Basic probability: in principle

1. Set preliminaries:
   (a) If S is a set, what do we mean by its ***power set*** $\mathscr{P}(S)$?
   (b) What, then, is the simple meaning of the statement that $A \in \mathscr{P}(S)$?
   (c) If S is finite, what is $|\mathscr{P}(S)|$?

2. When we use probability to study a system, briefly explain the difference between the ***sample space*** and the ***event space*** (illustrate with the examples of a coin-flip or rolling of a die).
   What is an ***event*** simply another name for?

3. Suppose that Ω is a finite sample space, with $|\Omega| = n \geq 1$.
   (a) What is a ***probability distribution*** on Ω, and how does this allow us to compute probabilities of *events*?
   (b) What is a ***uniform probability distribution*** on Ω, and what does this mean about:
      (i) The probability attached to each *sample* in $x \in \Omega$?
      (ii) The probability attached to each *event* in $A \subset \Omega$?

4. How can we approach any basic probability question in the case of a uniform probability distribution?

. . . and in practice

5. Consider the sample space of rolls of a pair of fair 6-sided dice (for simplicity, consider the dice to be distinct).
   How many possible pairs (m, n) of rolls are there? What is the probability of each one?
   Make a chart of all of the possible rolls, as an aid in counting for the parts below.
   Determine the probabilities of each of the following events:
   (a) Both dice show the same number.
   (b) The first die shows a strictly greater number than the second.
   (c) S_n, in which the *sum* of the numbers the dice show is n (compute these for $n = 1, 2, \ldots, 12$).

6. Consider the sample space of 10 consecutive flips of a fair coin (i.e., all sequences of `H` & `T` of length 10).
   What is the size of this sample space? What is the size of the corresponding event space?
   Determine the probabilities of each of the following events:
   (a) The first flip is `H`.
   (b) The first and second flips have identical results.
   (c) The first and second flips have different results.
   (d) All flips are `T`.
   (e) The first 9 flips are all `T`, and the tenth is `H`.
   (f) The flips are `HTHTHTHTHT`.
   (g) H_k, in which there are *exactly* k `H`'s in the list of rolls (compute these for $k = 0, 1, \ldots, 10$).
   (h) There are exactly 3 `H`'s in the first five rolls, then exactly 2 `H`'s in the next five rolls.
   How does this compare with the value of H_5 in the previous part?